ACCURATE AND EFFICIENT CAD-ORIENTED MODELING OF CIRCULAR WAVEGUIDE BENDS

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Abstract— **An extended local modes approach suitable for computer-aided design of curved waveguide bends is presented. This together with a new equivalent network representation of the circular bend consisting of uncoupled modes and coupling networks are found to provide accurate and efficient computer-aided design and simulation tools for circular bends.**

I. INTRODUCTION

CIRCULAR waveguide bends are frequently used in many microwave systems such as for satellite communications, radar, and other applications. Typical design requirements for waveguide bends include compactness of the waveguide network and low return loss over a wide frequency range. In addition, accurate knowledge and control of the phase of the transmission coefficient is often critical in the design of waveguide components such as phase shifters for beam forming networks [1]. Therefore, an accurate and efficient full-wave model for waveguide bends should be a part of a modern design tool for computer-aided design (CAD) of waveguide circuits and components.

Waveguide bends have been modeled by a number of workers during the past few decades (e.g. [2]-[10]. The earlier methods (e.g. [2] - [5]), however, are not suitable for use in a CAD tool to design complex waveguide circuits because of their limited accuracy or low efficiency. Recently, Weisshaar *et al.* developed an efficient full-wave technique which is based on a method of moments solution of the wave equation combined with a mode matching technique using the generalized scattering matrix formulation [6]. Since then, several other papers on accurate full-wave modeling of curved waveguide bends have been published ([7]-[10]).

Modal expansion methods such as [6], [9], [10] are particularly attractive for use in CAD tools because of their computational efficiency and physical insight. In addition, equivalent network formulations are often preferred in many CAD applications because of the ease of combining different circuit components and the potential of using a powerful circuit simulator engine, as demonstrated in [11]. To this end, [10] has reformulated the scattering matrix approach given in [6] in terms of the corresponding admittance and impedance matrix description. A disadvantage of the formulation in [6], [10], however, is the requirement of numerical integration. Numerical integration has been entirely eliminated in the related local modes approach [9]. As in the mode-matching technique, the transverse fields are expanded in terms of the locally straight waveguide modes which are not orthogonal in the curved bend region. The resulting mode coupling inside the bend is described by a system of generalized telegraphist's equations which is directly solved for the transmission matrix of the curved bend discontinuity. The efficiency of the approach could be enhanced by utilizing the modal spectrum of the curved bend.

In this paper, the local modes technique is extended to obtain the modal solutions in the curved region resulting in improved numerical efficiency in calculating the transmission characteristics. In addition, a normal mode theory based on coupled transmission lines [12] is applied to decouple the modes in the curved bend. The numerical efficiency of the local modes method is then compared to the mode-matching technique [6] for computation of both the modal characteristics and the scattering parameters.

II. THEORY

A. Modal Solutions by Local Modes Approach

Figure 1 shows the geometry of a circular H-plane bend of angle α in a rectangular waveguide. The transverse electric fields E_y and H_x are expanded in terms of the modes of the locally straight waveguides, i.e. the local modes. The transverse electric and magnetic fields in the curved waveguide are given by

$$\begin{aligned} E_y(x,\varphi) &= \sum_n V_n(\varphi)\phi_n(x) \\ H_x(x,\varphi) &= -\sum_n I_n(\varphi)\phi_n(x) \end{aligned} \tag{1}$$

where V_n and I_n are the total voltage and current amplitudes of mode n, and $\phi_n(x)$ are transverse field solutions of the locally straight waveguide. The generalized telegraphist's equations relating the voltage and current amplitude coefficients are obtained from a Galerkin's procedure, as described

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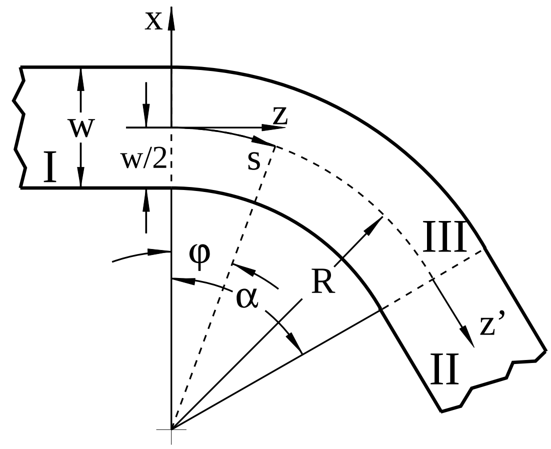

Fig. 1. Geometry of the curved bend discontinuity.

in [9]. The result can be expressed in compact matrix notation as

$$\begin{aligned} \partial_\varphi \mathbf{v} &= -\mathbf{P}\mathbf{i} \\ \partial_\varphi \mathbf{i} &= \mathbf{Q}\mathbf{v} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathbf{P} &= j\omega\mu_0(R_i\mathbf{U} + \mathbf{H}) \\ \mathbf{Q} &= \frac{1}{2(\omega\mu_0)^2}[\mathbf{D}_\beta\mathbf{P} + \mathbf{P}\mathbf{D}_\beta] \end{aligned} \tag{3}$$

Here, **U** is the unity matrix, $H_{mn} = \langle x\phi_m(x), \phi_n(x)\rangle$, and $\mathbf{D}_\beta = diag(\beta_n^2)$ where β_n are the phase constants of the modes of the locally straight waveguide.

Eq. (2) describes a set of coupled first order ordinary differential equations which represent the coupling of the modes of the locally straight waveguide in the curved waveguide region. It should be noted that the only integrations involved in (2) are in the matrix elements H_{mn} and can be given in simple closed form. On the other hand, the method of moments solution in [6], [10] obtained from the wave equation requires some numerical integrations which clearly is a disadvantage for implementation in a CAD tool.

In [9], the *ABCD* matrix for a bend with angle α is obtained in form of a matrix exponential. In order to avoid numerical instabilities due to evanescent modes, however, the bend is partitioned into many sections of sufficiently small angle. The small angular sections are then combined using the generalized scattering matrix technique. This is a further disadvantage for implementation of the local modes approach [9] in a CAD tool. Furthermore, the local mode formulation of [9] does not lead to a simple CAD compatible equivalent network representation due to the coupled transmission lines network in the bend region.

In the work presented here, (2) is solved for the eigenmodes (uncoupled modes) in the curved waveguide. As shown later, this formulation leads directly to an equivalent network formulation in terms of uncoupled modal transmission lines and coupling networks. The modal voltages $V_n(x,\varphi)$ and currents $I_n(x,\varphi)$ are assumed to be of the form

$$\begin{aligned} \partial_\varphi V_n &= \sum_m C^v_{mn}\tilde{\phi_m}(x)\exp\left(-j\tilde{\beta_m}R\varphi\right) \\ \partial_\varphi I_n &= \textstyle\sum_m C^i_{mn}\tilde{\phi_m}(x)\exp\left(-j\tilde{\beta_m}R\varphi\right) \end{aligned} \tag{4}$$

where $\tilde{\beta_m}$ are the phase constants of the uncoupled modes in the curved waveguide and C^v_{mn} and C^i_{mn} are the voltage and current expansion coefficients, respectively. Inserting (4) into (2) results in two equivalent eigenvalue matrix equations

$$\begin{aligned} (\tilde{\beta}R)^2\mathbf{v} &= \mathbf{P}\mathbf{Q}\mathbf{v} \\ (\tilde{\beta}R)^2\mathbf{i} &= \mathbf{Q}\mathbf{P}\mathbf{i} \end{aligned} \tag{5}$$

with eigenvalue matrix $D_{\tilde{\beta}} = diag(\tilde{\beta_n}^2R^2)$ and voltage eigenvector matrix $\{C^v_{mn}\}$ and current eigenvector matrix $\{C^i_{mn}\}$ where $\mathbf{C}_v = (\mathbf{C}_i^T)^{-1}$. Similar to the normal mode theory for coupled transmission lines [12], it can be shown that the voltage and current eigenvector matrices diagonalize matrices **P** and **Q** given in (3) as

$$\begin{aligned} \mathbf{C}_v\mathbf{P}(\mathbf{C}_v^T)^{-1} &= diag(j\tilde{\beta}_n\tilde{Z}_n) \\ \mathbf{C}_v^T\mathbf{Q}\mathbf{C}_v &= diag(j\tilde{\beta}_n\tilde{Y}_n) \end{aligned} \tag{6}$$

The modal phase constants $\tilde{\beta}_n$ and modal impedances $\tilde{Z}_n = 1/\tilde{Y}_n$ can be obtained from the product and quotient of the eigenvalues of (6), respectively. The diagonalization of $\mathbf{P}$ and $\mathbf{Q}$ leads to the decomposition of the curved bend into (uncoupled) modal transmission lines described by

$$\begin{aligned} \partial_{(R\varphi)}\mathbf{e} &= -diag(j\tilde{\beta}_n\tilde{Z}_n)\mathbf{h} \\ \partial_{(R\varphi)}\mathbf{h} &= -diag(j\tilde{\beta}_n\tilde{Y}_n)\mathbf{e} \end{aligned} \tag{7}$$

and coupling networks described by

$$\begin{aligned} \mathbf{v} &= \mathbf{C}_v\mathbf{e} \\ \mathbf{h} &= \mathbf{C}_v^T\mathbf{i} \end{aligned} \tag{8}$$

Similar to [11], (7) and (8) can be represented in terms of the equivalent network shown in Fig. 2. It should be noted that unlike the case of the step discontinuities, the coupling coefficients C^v_{mn} are frequency-dependent because of the frequency-dependence of the modal fields, as illustrated in Fig. 3.

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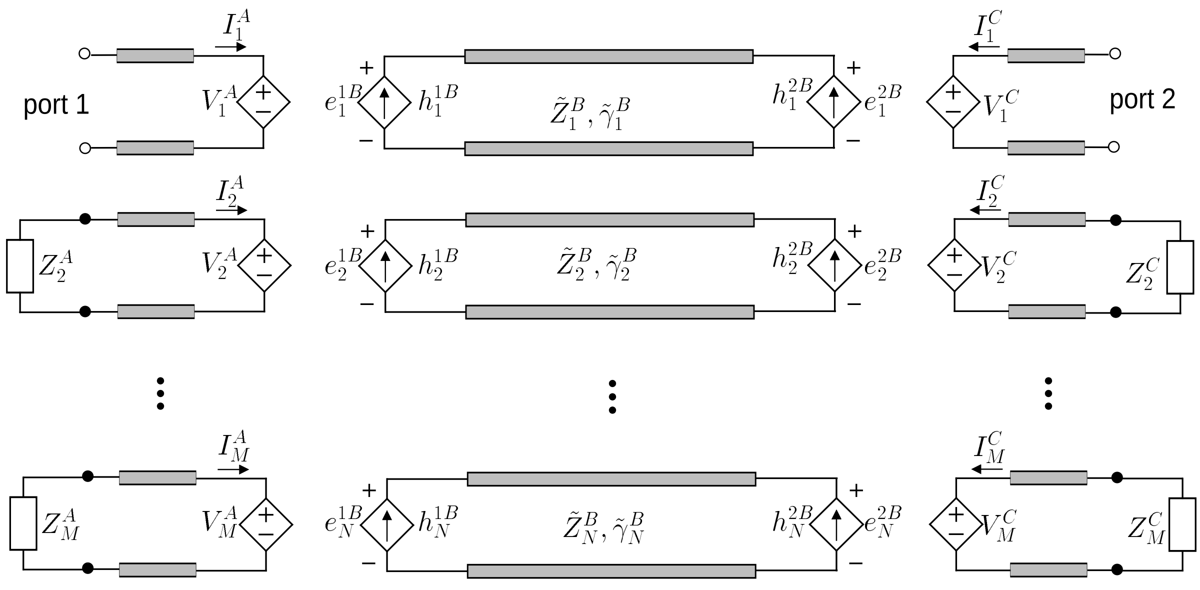

Fig. 2. Multi-mode equivalent network representation of a circular waveguide bend discontinuity.

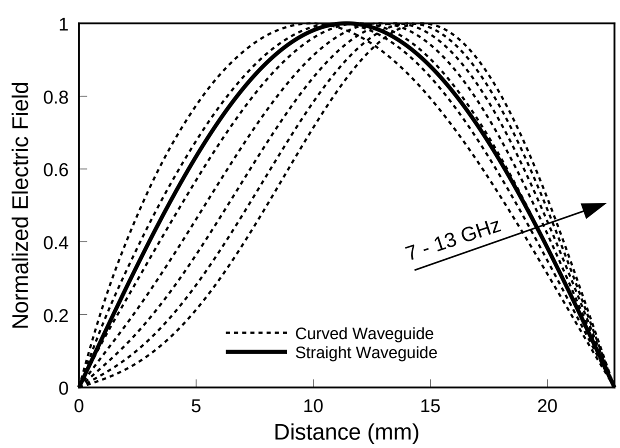

Fig. 3. Frequency-dependence of the electric field of the dominant mode in a curved bend in a WR90 waveguide ($a = 22.86$ mm, $R = 15$ mm).

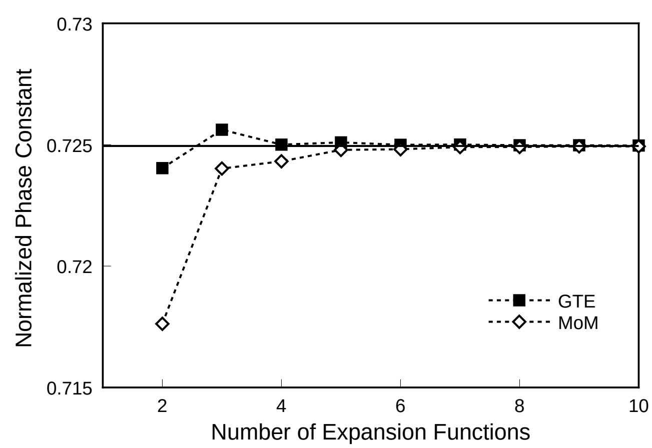

Fig. 4. Convergence behavior of the phase constant of the dominant mode of a circular H-plane bend in a WR90 waveguide ($R = 12$ mm, $a = 22.86$ mm, $f = 10$ GHz).

III. Results

Extensive simulations of the modal characteristics and scattering parameters of curved bend discontinuities in a rectangular waveguide have been performed. Fig. 4 shows a typical convergence plot for the normalized phase constant in a curved H-plane bend for both the local modes (GTE - generalized telegraphist's equations) approach and the method of moments approach described in [6]. It is found that the local modes solution converges somewhat faster than the method of moments solution for large bend radii and in the middle and upper frequency band.

The convergence behavior of the scattering parameters of the curved bend discontinuity is illustrated in Fig. 5. It is found from this and other simulation results that both methods exhibit similar convergence behavior with a small advantage for the method of moments approach over the local modes approach. This is attributed to a better convergence of the field expansion with the method of moments and is consistent with similar results found from a comparison of the method of moments and generalized telegraphist's solutions for planar dielectric waveguides [13].

IV. Conclusion

An extended local modes approach for curved bends in a rectangular waveguide has been presented. It was demon-

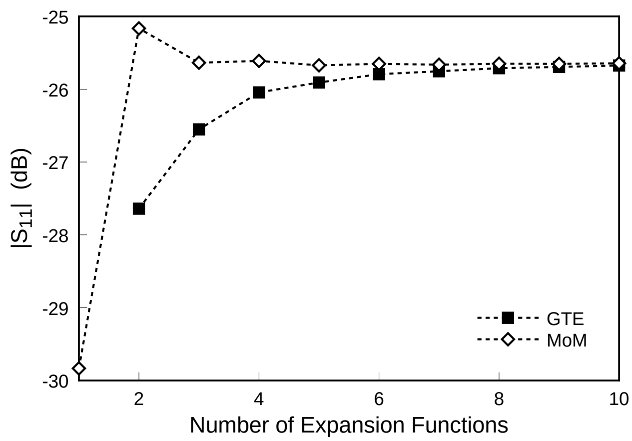

Fig. 5. Reflection coefficient of a circular H-plane bend discontinuity in a WR90 waveguide ($R = 12$ mm, $a = 22.86$ mm, $f = 10$ GHz).

strated that this approach provides the modal solutions and transmission characteristics of a curved bend with accuracy similar to the mode-matching approach for a given number of expansion functions, but avoids any numerical integration. In addition, an equivalent network representation of the circular bend consisting of uncoupled normal modes and coupling networks has been presented. The network representation is identical in form to that of a thick iris in a rectangular waveguide. However, as pointed out, the coupling coefficients are frequency-dependent because of the frequency-dependence of the modal field solution in the curved bend region. This frequency-dependence is unavoidable and increases the computational burden with any technique.

The proposed technique should be well-suited for CAD applications as an accurate and efficient characterization tool of curved bends. The technique and corresponding results presented here do not only apply to rectangular waveguides but also to other guided-wave structures such as microstrip bends in microwave integrated circuits (e.g. [14]). The equivalent network representation is expected to provide more flexibility in the computer-aided simulation and design of complex systems containing curved bends.

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